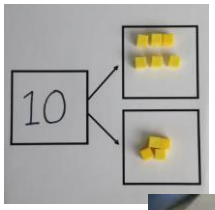
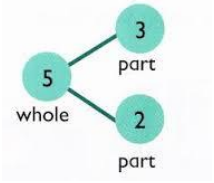
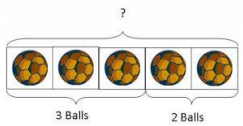
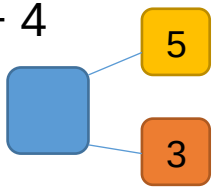
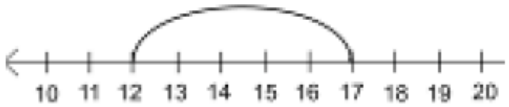
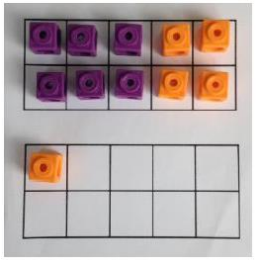
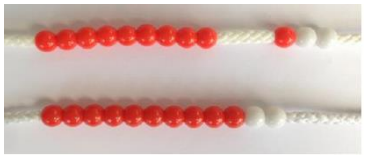
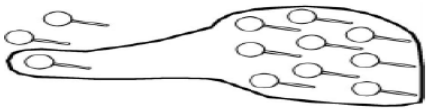
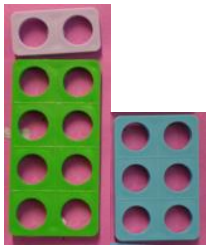
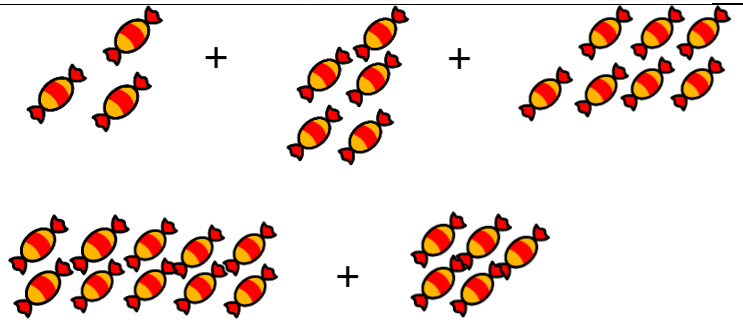
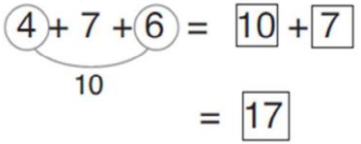
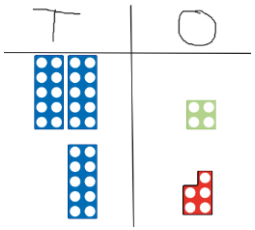
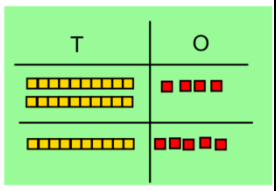
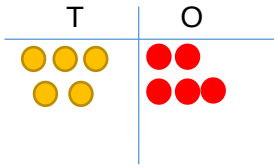
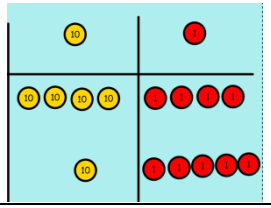
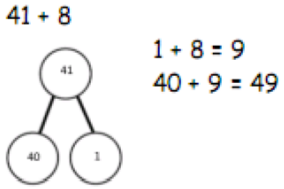
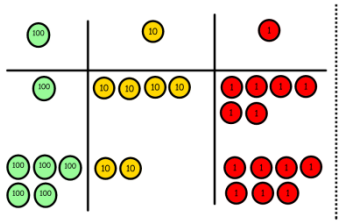
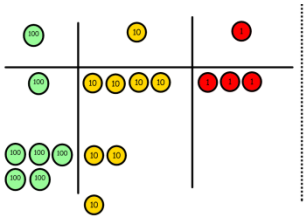
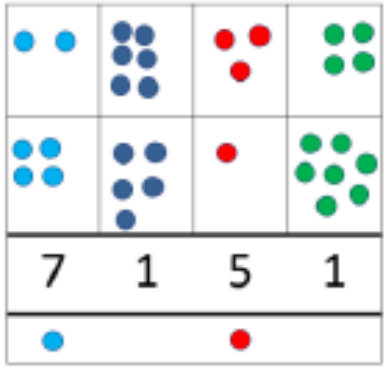
Wouldham All Saints Primary School

Progression in Calculations

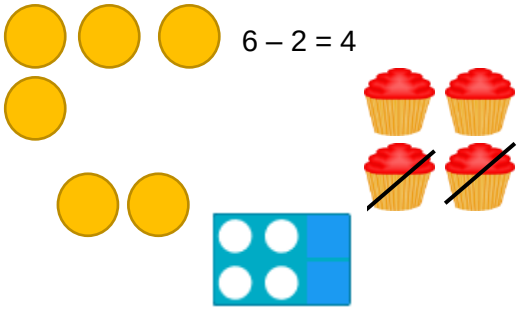
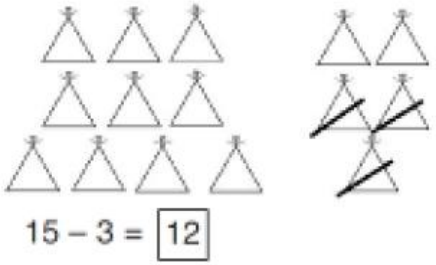
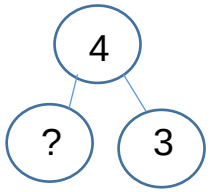
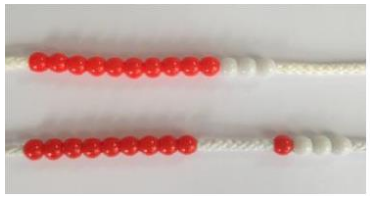
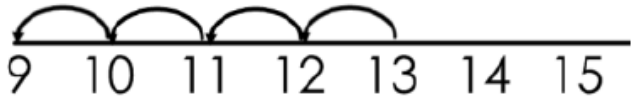
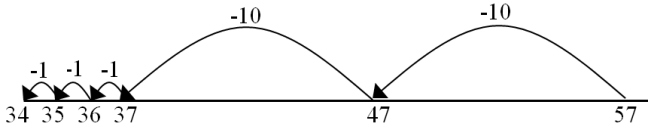
Addition

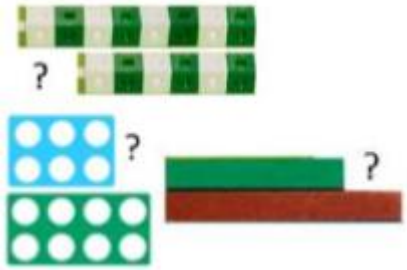
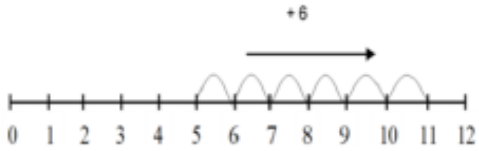
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use manipulatives to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number and counting on using Numicon, counters, beads, cubes, base 10. 	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.	 6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. 	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 9 + 5 = 14 1 4 +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	Using Numicon or Cuisenaire rods. 8 + 6 + 2 = 16 Put 8 and 2 together to make 10. Add on 6.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use Numicon first before moving on to base 10 blocks and place value counters.  	After practically using the base 10 blocks, children can draw the blocks to help them to solve additions.  	Expanded column addition to reinforce place value understanding. 21 + 42 = 20 + 1 40 + 2 ---------- 60 + 3 = 63

			
Column method- regrouping	Make both numbers on a place value grid using base 10.  146 + 527 Add up the units and exchange 10 ones for one 10.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 1 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

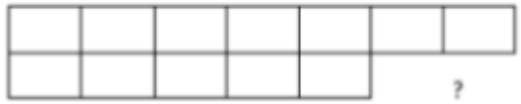
Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes Numicon and subtraction covers to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away. 	Vary layout of calculations 18 - 3= 15 8 – 2 = 6 __ = 4 – 3 
Counting back	Use counters and move them away from the group as you take them away counting backwards as you go.  Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

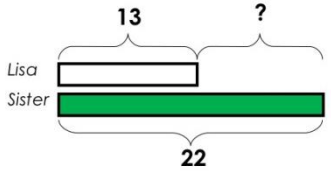
<table>
<tr>
<td>Find the difference</td>
<td>Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used).
 
</td>
<td> 

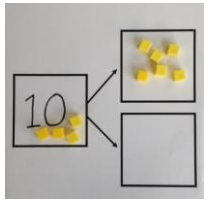
Count on to find the difference.

Children to draw the cubes/other concrete objects which they have used
XXXXXXXX
XXXXXX

Use of the bar model
 

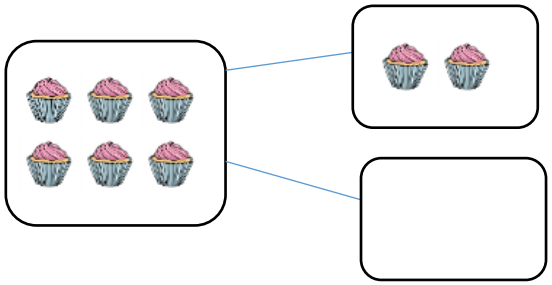
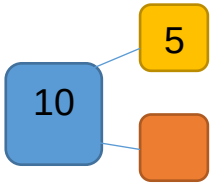
Draw bars to find the difference between 2 numbers.

Lisa is 13 years old. Her sister is 22 years old.
Find the difference in age between them.
 
</td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

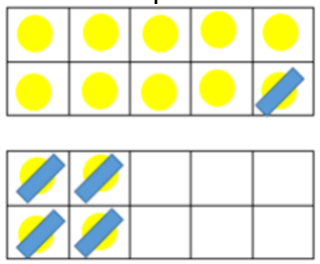
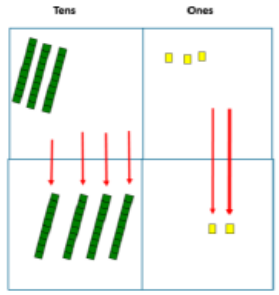
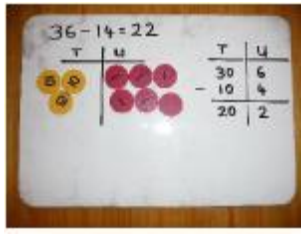
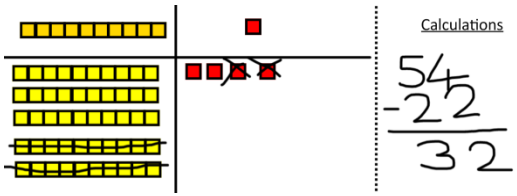
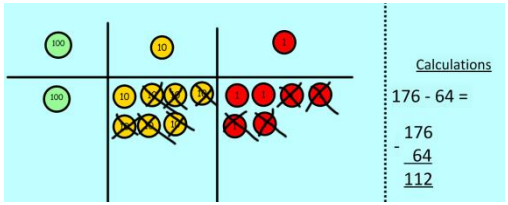
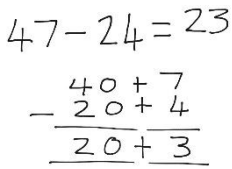
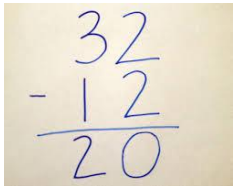
Children to also explore why 9-7 = 8-6 (the difference, of each digit, by 1 so the difference is the same)</td>
</tr>
<tr>
<td>Part Part Whole Model</td>
<td> 

Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

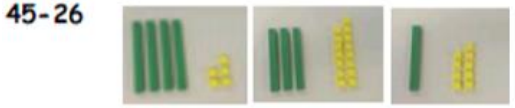
10 - 6 =</td>
<td>Use a pictorial representation of objects/numicon to show the part part whole model.
  </td>
<td> 

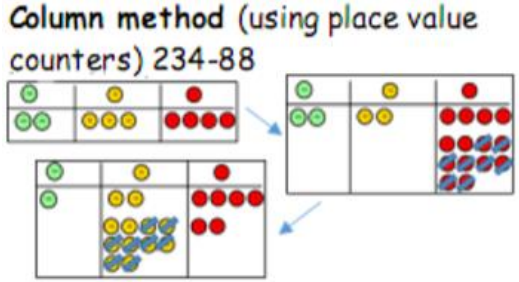
Move to using numbers within the part whole model.</td>
</tr>
</table>

Make 10	Using numicon or ten frames 14 – 5 =  Children could also do this by subtracting 5 from 10. 	Children to present the ten frame pictorially. 	Children to use their knowledge of related facts eg. 14 – 5 = I know that if I take 4 I will get to 10, then there is just 1 more to take away.
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	  Draw the Base 10 or place value counters alongside the written calculation to help to show working.	 This will lead to a clear written column subtraction. 

Column method with regrouping

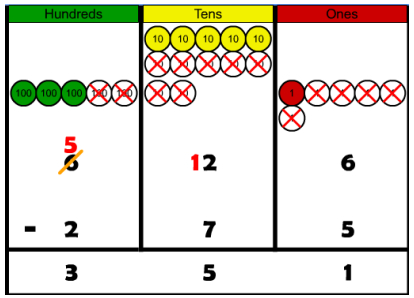
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.





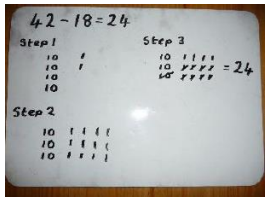
Exchange place value counters before taking away.

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

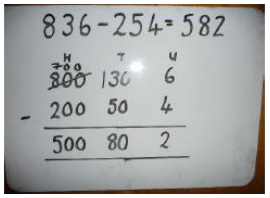


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

When confident, children can find their own way to record the exchange/regrouping e.g.

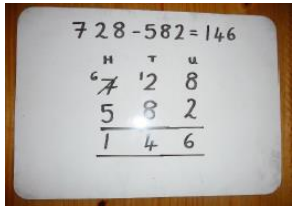


Children can start their formal written method by partitioning the number into clear place value columns.

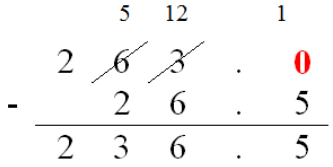


Children need to need to understand the importance of the place value holder 0.

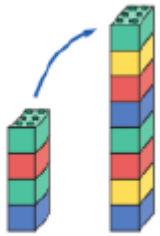
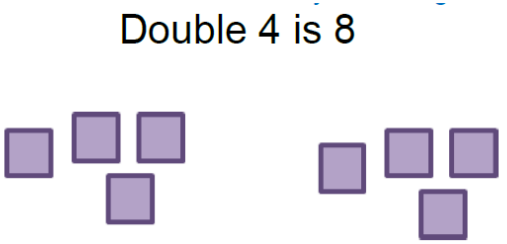
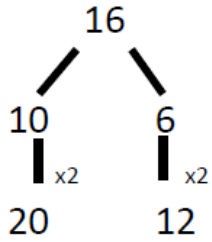
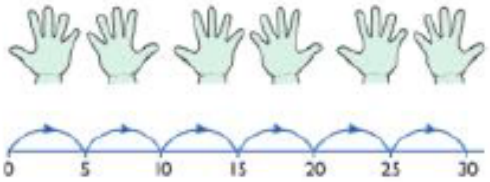
Moving forward the children use a more compact method.

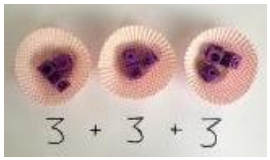
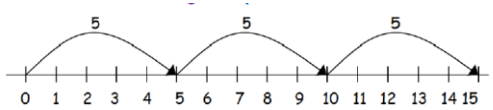
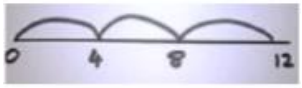
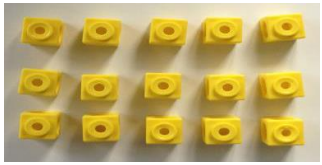
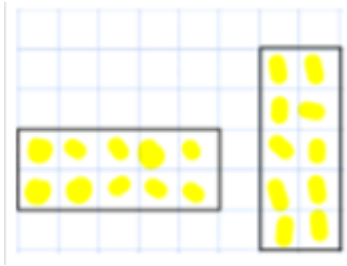


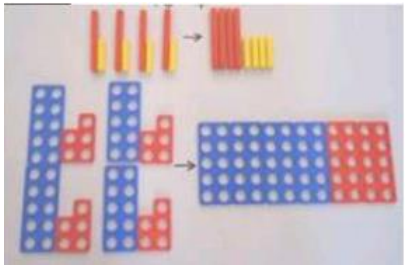
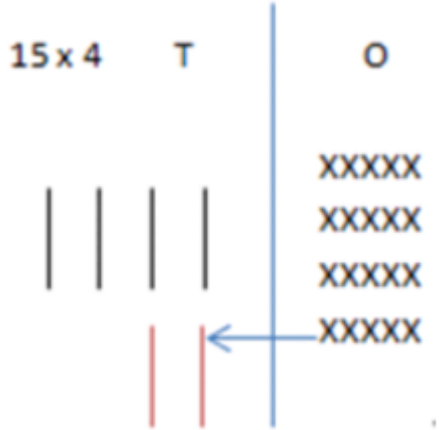
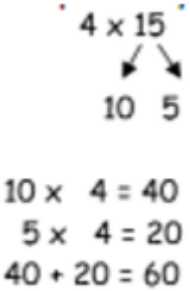
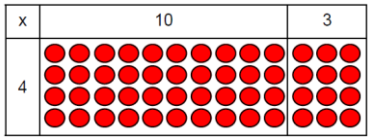
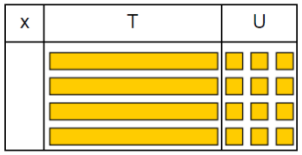
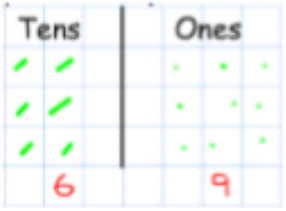
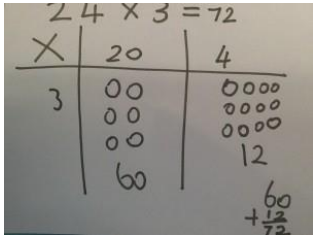
This will lead to an understanding of subtracting any number including decimals.

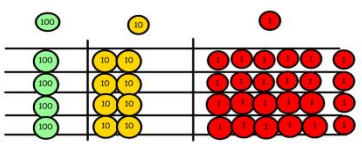
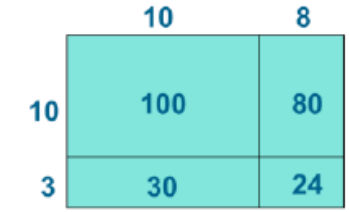
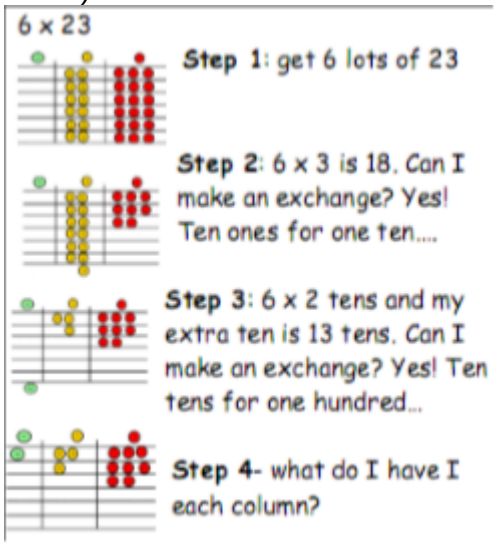
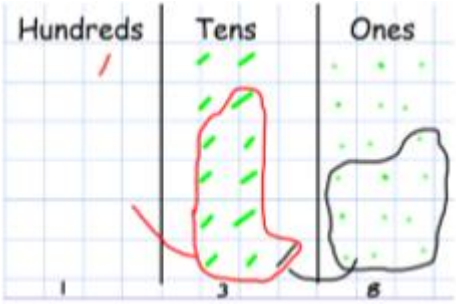


Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples	   Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition	  Use different objects to add equal groups. 	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  If appropriate this may include shown on a blank number line. 
Arrays- showing commutative multiplication	Create arrays using counters/cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  2 x 5 = 10 5 x 2 = 10	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Partition to multiply	Use numicon, base 10, Cuisenaire rods 4 x 15 = 	Children to represent the concrete manipulatives ina picture eg base 10 can be represented like: 	Children to be encouraged to show the steps they have taken. 
Grid Method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows. Fill each row with 126.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. 3 x 23 =  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.  210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	 Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.		 10 8 10 100 80 3 30 24 X | 1000 | 300 | 40 | 2 10 | 10000 | 3000 | 400 | 20 8 | 8000 | 2400 | 320 | 16
Column multiplication	Begin with using place value counters (children need this stage initially, to understand how the column method works)  6 x 23 **Step 1**: get 6 lots of 23 **Step 2**: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten.... **Step 3**: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred... **Step 4**- what do I have I each column?	Children to represent the counters/base 10 pictorially eg.  Hundreds Tens Ones 1 3 8	The aim is to get to the formal method but children need to understand how it works. 6 x 23 = 23 × 6 138 1 1

Long Multiplication

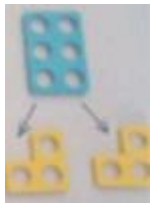
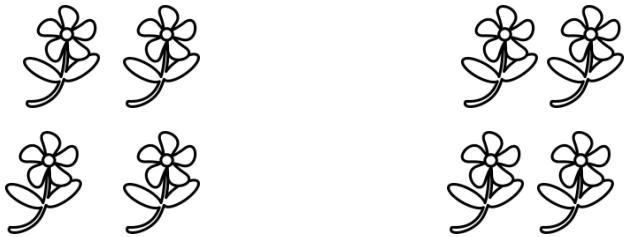
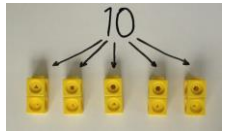
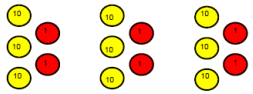
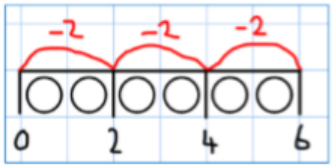
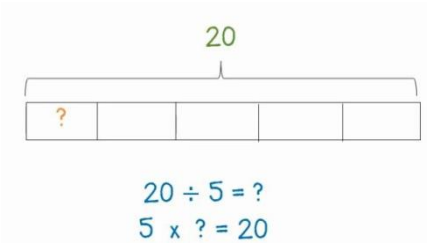
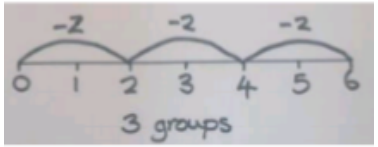
Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

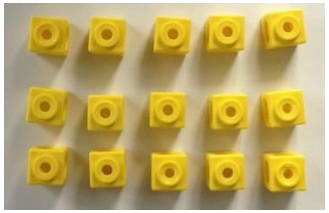
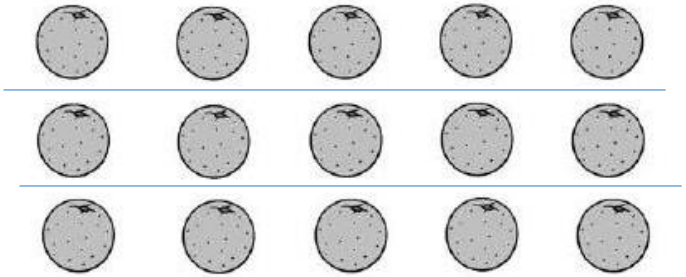
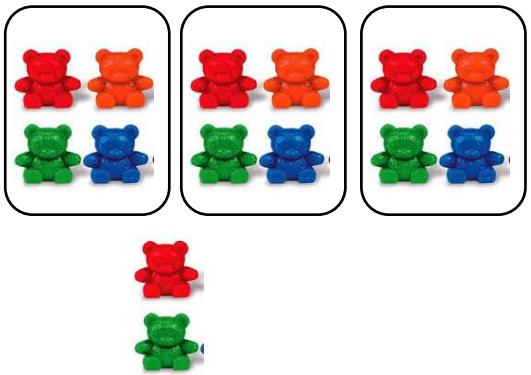
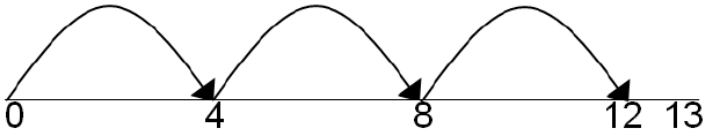
Children to write out what they are solving next to their answer.

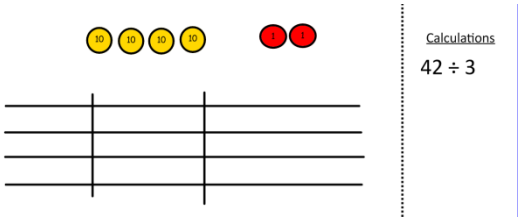
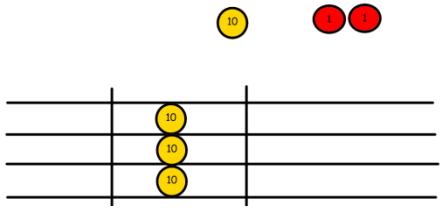
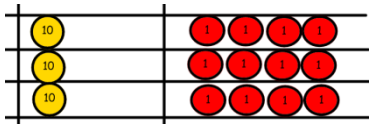
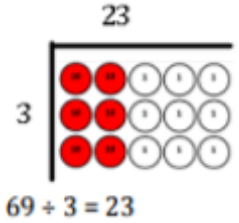
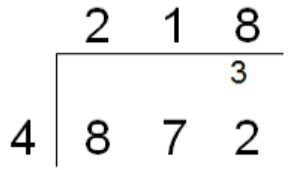
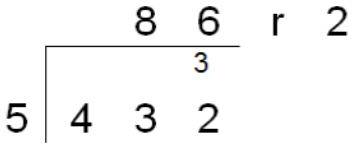
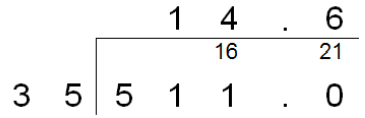
```
  32
x 24
  ---
   8    (4 x 2)
 120    (4 x 30)
  40    (20 x 2)
 600    (20 x 30)
 ---
 768
```

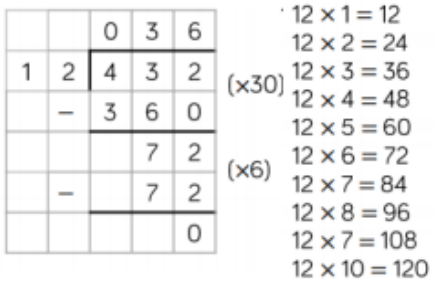
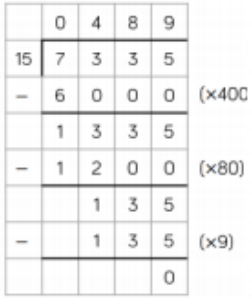
This moves to the more compact method

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups?  	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, numicon, objects or place value counters to aid understanding.    96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	Abstract number line 

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Children can represent the resources that they have used pictorally e.g.  Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 	Complete written divisions and show the remainder using r.  Children need to be able to decide what to do with a remainder in the context of a problem.

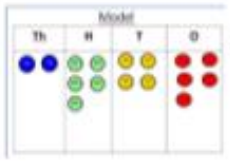
Short division	Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Extending division to resemble written method of short division using jottings to aid calculationa nd demonstarte understanding.  	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  Children should also be able to represent remainders as fractions.

Long Division by chunking	When children begin to divide up to 4-digits by 2-digits, written methods become most accurate as concrete and pictoral representations become less effective. Children can write out multiples to suppoer their calculations with larger remainders.	 12 × 1 = 12 12 × 2 = 24 12 × 3 = 36 12 × 4 = 48 12 × 5 = 60 12 × 6 = 72 12 × 7 = 84 12 × 8 = 96 12 × 7 = 108 12 × 10 = 120 **432 ÷ 12 = 36** **7,335 ÷ 15 = 489**  1 × 15 = 15 2 × 15 = 30 3 × 15 = 45 4 × 15 = 60 5 × 15 = 75 10 × 15 = 150

Long Division

Children should also be taught this method so that they can choose the method that they prefer to use.

Use place value counters and pictures to represent the long division method for dividing a number by 2 digits:

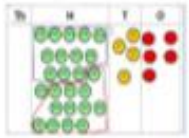


$$\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

2544 ÷ 12

How many groups of 12 thousands do we have? None

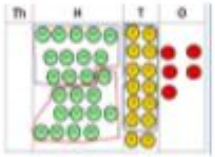
Exchange 2 thousand for 20 hundreds.



$$\begin{array}{r} 0\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$

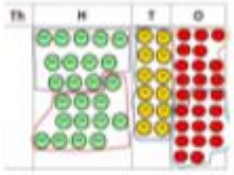
How many groups of 12 are in 25 hundreds? 2 groups. Circle them.

We have grouped 24 hundreds so can take them off and we are left with one.



$$\begin{array}{r} 0\,2\,1 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

Key language – 'how many groups of x can we make with x hundreds'

$$\begin{array}{r} 0 \\ 12\overline{)2544} \end{array}$$

Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

$$\begin{array}{r} 0\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

$$\begin{array}{r} 0\,2\,1 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens?
The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.

$$\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

Moving on to decimals

432 ÷ 15 becomes

$$\begin{array}{r} 2\,8\cdot 8 \\ 15\overline{)4\,3\,2\cdot 0} \\ 3\,0 \\ \hline 1\,3\,2 \\ 1\,2\,0 \\ \hline 1\,2\,0 \\ 1\,2\,0 \\ \hline 0 \end{array}$$

Answer: 28·8